FastTrack - MA109
Rational Expressions

Katherine Paullin, Ph.D.
Lecturer, Department of Mathematics
University of Kentucky
katherine.paullin@uky.edu

Wednesday, August 17, 2016

Outline

Reduce the fraction.

$$\frac{24}{40}$$

Add

$$\frac{1}{7} + \frac{4}{5}$$

Section 1

Writing a Rational Expression in Simplest Form

Simplest Form

A rational expression is in **simplest form** when the numerator and denominator have no common factors (other than 1). After factoring the numerator and denominator, we apply the **fundamental property of rational expressions**.

Fundamental Property of Rational Expressions

If P, Q, and R are polynomials, with $Q, R \neq 0$,

$$\frac{P \cdot R}{Q \cdot R} = \frac{P}{Q} \text{ AND } \frac{P}{Q} = \frac{P \cdot R}{Q \cdot R}$$

Examples

Write the Expression in Simplest form:
$$\frac{x^2 - 1}{x^2 - 3x + 2}$$

Factor, Factor, Factor!!

$$\frac{x^2 - 1}{x^2 - 3x + 2} = \frac{(x-1)(x+1)}{(x-1)(x-2)}$$
$$= \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x-2)}$$
$$= \frac{(x+1)}{(x-2)}$$

You MUST cancel entire factors, not pieces of them.

WARNING! Do NOT do this:

$$\frac{x+1}{x+2} \neq \frac{\cancel{x}+1}{\cancel{x}+2} \neq \frac{1}{2}$$

Examples

Write the expression in simplest form.

1. $\dfrac{a-b}{b-a}$

$$\frac{a-b}{b-a} = \frac{-1(b-a)}{b-a} = \frac{-1\cancel{(b-a)}}{\cancel{b-a}}$$
$$= -1$$

2. $\dfrac{(6-2x)}{x^2-9}$

$$\frac{(6-2x)}{x^2-9} = \frac{2(3-x)}{(x-3)(x+3)} = \frac{2(3-x)}{(x-3)(x+3)}$$
$$= \frac{(2)(-1)}{x+3} = \frac{-2}{x+3}$$

Section 2

Multiplication and Division of Rational Expressions

Multiplying Rational Expressions

Given that $P, Q, R,$ and S are polynomials with $Q, S \neq 0$,

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$$

1. Factor all numerators and denominators completely.

2. Reduce common factors.

3. Multiply numerator $\times$ numerator and denominator $\times$ denominator.

Dividing Rational Expressions

Given that $P, Q, R,$ and S are polynomials with $Q, R, S \neq 0$,

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{PS}{QR}$$

Invert the divisor and multiply.

Examples

Compute the product.

$$\frac{2a+2}{3a-3a^2} \cdot \frac{3a^2-a-2}{9a^2-4}$$

$$\frac{2a+2}{3a-3a^2} \cdot \frac{3a^2-a-2}{9a^2-4} = \frac{2(a+1)}{3a(1-a)} \cdot \frac{(3a+2)(a-1)}{(3a+2)(3a-2)}$$

$$= \frac{2(a+1)}{3a(1-a)} \cdot \frac{\cancel{(3a+2)}\cancel{(a-1)}^{(-1)}}{\cancel{(3a+2)}(3a-2)}$$

$$= \frac{-2(a+1)}{3a(3a-2)}$$

Examples

Compute the quotient.

$$\frac{4m^3 - 12m^2 + 9m}{m^2 - 49} \div \frac{10m^2 - 15m}{m^2 + 4m - 21}$$

$$\frac{4m^3 - 12m^2 + 9m}{m^2 - 49} \div \frac{10m^2 - 15m}{m^2 + 4m - 21}$$

$$= \frac{4m^3 - 12m^2 + 9m}{m^2 - 49} \cdot \frac{m^2 + 4m - 21}{10m^2 - 15m}$$

$$= \frac{m(2m - 3)(2m - 3)}{(m + 7)(m - 7)} \cdot \frac{(m + 7)(m - 3)}{5m(2m - 3)}$$

$$= \frac{m(2m - 3)\cancel{(2m - 3)}}{\cancel{(m + 7)}(m - 7)} \cdot \frac{\cancel{(m + 7)}(m - 3)}{5m\cancel{(2m - 3)}}$$

$$= \frac{(2m - 3)(m - 3)}{5(m - 7)}$$

Section 3

Addition and Subtraction of Rational Expressions

Adding and Subtracting

Addition and Subtraction of Rational Expressions

1. Find the LCD of all rational expressions.
2. Build equivalent expressions using the LCD.
3. Add or subtract numerators as indicated.
4. Write the result in lowest terms.

Examples

Compute.

$$\frac{7}{10x} + \frac{3}{25x^2}$$

The LCD for $10x$ and $25x^2$ is $50x^2$.

$$\frac{7}{10x} + \frac{3}{25x^2} = \frac{7}{10x} \cdot \frac{5x}{5x} + \frac{3}{25x^2} \cdot \frac{2}{2}$$
$$= \frac{35x}{50x^2} + \frac{6}{50x^2}$$
$$= \frac{35x + 6}{50x^2}$$

Examples

Compute.

$$\frac{10x}{x^2 - 9} - \frac{5}{x - 3}$$

The LCD for $(x^2 - 9)$ and $(x - 3)$ is $(x - 3)(x + 3)$.

$$\frac{10x}{x^2 - 9} - \frac{5}{x - 3} = \frac{10x}{(x - 3)(x + 3)} - \frac{5}{x - 3} \cdot \frac{(x + 3)}{(x + 3)}$$

$$= \frac{10x - 5(x + 3)}{(x - 3)(x + 3)}$$

$$= \frac{5x - 15}{(x - 3)(x + 3)}$$

$$= \frac{5(x - 3)}{(x - 3)(x + 3)} = \frac{5\cancel{(x - 3)}}{\cancel{(x - 3)}(x + 3)}$$

$$= \frac{5}{x + 3}$$

Section 4

Simplifying Compound Fractions

Compound Fractions

A **compound fraction** is a rational expression whose numerator or denominator contain a fraction.

Example: $\dfrac{\frac{2}{3m} - \frac{3}{2}}{\frac{3}{4m} - \frac{1}{3m^2}}$

Simplifying Compound Fractions

Simplifying Compound Fractions (Method 1)

1. Add/subtract fractions in the numerator, writing them as a single expression.
2. Add/subtract fractions in the denominator, also writing them as a single expression.
3. multiply the numerator by the reciprocal of the denominator and simplify if possible.

Simplifying Compound Fractions (Method 2)

1. Find the LCD of all fractions in the numerator and the denominator.
2. Multiply the numerator and denominator by this LCD and simplify.
3. Simplify further if possible.

Examples

Simplify.

$$\frac{\frac{2}{3m} - \frac{3}{2}}{\frac{3}{4m} - \frac{1}{3m^2}}$$

The LCD for $3m, 2, 4m$, and $3m^2$ is $12m^2$.

$$\frac{\frac{2}{3m} - \frac{3}{2}}{\frac{3}{4m} - \frac{1}{3m^2}} = \frac{\left(\frac{2}{3m} - \frac{3}{2}\right)\left(\frac{12m^2}{1}\right)}{\left(\frac{3}{4m} - \frac{1}{3m^2}\right)\left(\frac{12m^2}{1}\right)}$$

$$= \frac{\left(\frac{2}{3m}\right)\left(\frac{12m^2}{1}\right) - \left(\frac{3}{2}\right)\left(\frac{12m^2}{1}\right)}{\left(\frac{3}{4m}\right)\left(\frac{12m^2}{1}\right) - \left(\frac{1}{3m^2}\right)\left(\frac{12m^2}{1}\right)}$$

$$= \frac{8m - 18m^2}{9m - 4}$$

$$= \frac{2m(4 - 9m)}{9m - 4} = \frac{2m\cancel{(4 - 9m)}^{(-1)}}{\cancel{9m - 4}}$$

$$= -2m$$

Section 5

Practice

Practice

1. Reduce to lowest terms. $\frac{5p^2-14p-3}{5p^2+11p+2}$

2. Compute $\frac{6v^2+23v+21}{4v^2-4v-15} \cdot \frac{4v^2-25}{3v+7}$

3. Compute $\frac{5b-10}{7b-28} \div \frac{2-b}{5b-20}$

1. Compute $\frac{5b-10}{7b-28} \div \frac{2-b}{5b-20}$

Practice - SOLUTIONS

1. Reduce to lowest terms. $\frac{5p^2-14p-3}{5p^2+11p+2}$

   $\frac{p-3}{p+2}$

2. Compute $\frac{6v^2+23v+21}{4v^2-4v-15} \cdot \frac{4v^2-25}{3v+7}$

   $2v+5$

3. Compute $\frac{5b-10}{7b-28} \div \frac{2-b}{5b-20}$

   $\frac{-25}{7}$

Practice

1. Subtract. $\dfrac{4n}{n^2 - 5n} - \dfrac{3}{4n - 20}$

2. Simplify the compound rational expression. $\dfrac{\frac{2}{y^2 - y - 20}}{\frac{3}{y+4} - \frac{4}{y-5}}$

Practice - SOLUTIONS

1. Subtract. $\dfrac{4n}{n^2 - 5n} - \dfrac{3}{4n - 20}$

   $\dfrac{13}{4(n - 5)}$

2. Simplify the compound rational expression. $\dfrac{\frac{2}{y^2 - y - 20}}{\frac{3}{y+4} - \frac{4}{y-5}}$

   $\dfrac{-2}{y + 31}$